数学Ⅰ・A 第1問〔1〕

(1) $ab=\dfrac{1+\sqrt{3}}{1+\sqrt{2}}\cdot\dfrac{1-\sqrt{3}}{1-\sqrt{2}}=\dfrac{1-3}{1-2}=^{ア}2$

$a+b=\dfrac{1+\sqrt{3}}{1+\sqrt{2}}+\dfrac{1-\sqrt{3}}{1-\sqrt{2}}=\dfrac{(1+\sqrt{3})(1-\sqrt{2})+(1-\sqrt{3})(1+\sqrt{2})}{(1+\sqrt{2})(1-\sqrt{2})}$

$=\dfrac{1+\sqrt{3}-\sqrt{2}-\sqrt{6}+1-\sqrt{3}+\sqrt{2}-\sqrt{6}}{1-2}=^{イ}2(^{ウエ}-1+\sqrt{^{オ}6})$

$a^2+b^2=(a+b)^2-2ab=\{2(-1+\sqrt{6})\}^2-2\cdot2=4(7-2\sqrt{6})-4$

$=4(6-2\sqrt{6})=^{カ}8(^{キ}3-\sqrt{^{ク}6})$

(2) (1)から　$a^2+b^2+4(a+b)=8(3-\sqrt{6})+4\cdot2(-1+\sqrt{6})=^{ケコ}16$

$ab=2$ から　$b=\dfrac{2}{a}$

これを $a^2+b^2+4(a+b)=16$ に代入して　$a^2+\left(\dfrac{2}{a}\right)^2+4\left(a+\dfrac{2}{a}\right)=16$

両辺に a^2 を掛けて整理すると　$a^4+^{サ}4a^3-^{シス}16a^2+^{セ}8a+^{ソ}4=0$